Modelling and predicting extreme behavior in Critical Real-Time systems with Advanced Statistics

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I. Extended Abstract

In the last decade, the market for Critical Real-Time Embedded Systems (CRTES) has increased significantly. According to Global Markets Insight [1], the embedded systems market will reach a total size of US $258 billion in 2023 at an average annual growth rate of 5.6%. Their extensive use in domains such as automotive, aerospace and avionics industry demands ever increasing performance requirements [2]. To satisfy those requirements the CRTES industry has implemented more complex processors, a higher number of memory modules, and accelerators units. Thus the demanding performance requirements have led to a merge of CRTES with High Performance systems. All of these industries work within the framework of CRTES, which puts several restrictions in their design and implementation. Real Time systems require to deliver a response to an event in a restricted time frame or deadline. Real-time systems where missing a deadline provokes a total system failure (hard real-time systems) need satisfy certain guidelines and standards to show that they comply with test for functional and timing behaviour. These standards change depending on the industry, for instance the automotive industry follows ISO 26262 [3] and the aerospace industry follows DO-178C [4]. Researches have developed techniques to analyse the timing correctness in a CRTES. Here, we will expose how they perform on the estimation of the Worst-Case Execution Time (WCET). The WCET is the maximum time that a particular software takes to execute. Estimating its value is crucial from a timing analysis point of view. However there is still not a generalised precise and safe method to produce estimates of WCET [5]. In the CRTES the estimations of the WCET cannot be lower than the true WCET, as they are deemed unsafe; but they cannot exceed it by a significant margin, as they will be deemed pessimistic and impractical.

There are two main frameworks for Timing Analysis. On the one hand there is Static Timing Analysis, which aims at developing an analytical model of the hardware in order to compute the timing of a program. The complexity of the hardware nowadays has increased and Real-Time systems in these days have operating manuals of the order of thousands of pages. If one assumes that they contain complete trustworthy information, it is still a herculean task. On the other hand researches have resorted to a measurement-based approach, called Measurement Based Timing Analysis (MBTA). Here, the timing analysis is deduced from traces of the execution time. The WCET is deduced from the empirical distribution of the data. In fact the commonly used reference value to obtain is the probabilistic WCET (pWCET). In probabilistic analysis one wants to obtain the probability of exceeding a certain value, and the tool to obtain it lies in Extreme Value Theory [6]. Thus, the pWCET is not a single value like the WCET. Instead it is a distribution function that computes the probability of exceeding high quantiles. This encapsulates the essence of Measurement Based Probabilistic Timing Analysis (MBPTA) [7], [8], [9]. It separates from MBTA in that the empirical distribution of the execution time of multiple experiments, is used to compute a probabilistic WCET (pWCET). This methodology has received the support to be compliant with the safety standards. The aim here is not to estimate the exact WCET, but to compute a distribution that estimates the high quantiles of the empirical distribution of the experiments. All techniques, including the static one need to satisfy two properties. First and foremost, the estimated WCET cannot be lower than the true WCET. Hard real-time systems are design to meet the deadline, otherwise a failure of the system occurs. It is not safe to obtain an optimistic estimation of the maximum delay possible as it leads to potentially dangerous situations. Secondly, the estimated WCET cannot be too far from the true WCET. Determining a resource budget for the software is s big part of critical systems, otherwise it could lead to inefficient and expensive systems.

Extreme Value Theory (EVT) has been the theoretical framework to work with MBPTA and specifically in estimating pWCET. EVT deals with the extreme deviations of the data and provides with tools to contextualize and estimate their behaviour. The tail, the extreme ends of the distribution are characterised based on the value of the extreme value index (evi). Tails lighter than exponential ones (so with $evi < 0$) can deliver tighter bounds, as discussed in [10]. Yet, in the context of EVT, either GEV or GPD, distributions with $evi < 0$ have a compact support, i.e. they have an absolute maximum value that cannot be exceeded. Hence, light tails in the case of EVT have an intrinsic risk of delivering optimistic tail distributions. As we did in our work in [11], we overcame the limitation of the data and delivering a practical solution to obtain pWCET estimates tighter than those of exponential tails while preserving reliability. We did so by complementing EVT with survivability analysis as the theoretical ground for our hypothesis. One of the traits of CRTES is that programs need to finish. They have a maximum budget for time that cannot be exceeded for energy and safety reasons. This translated

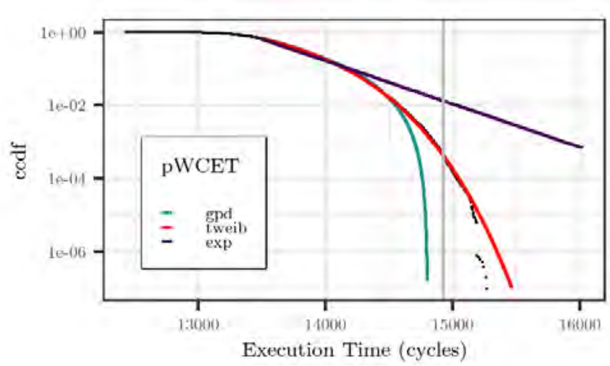

Fig. 2: Whole data of $n = 10^7$ of railway case study data with different pWCET fittings

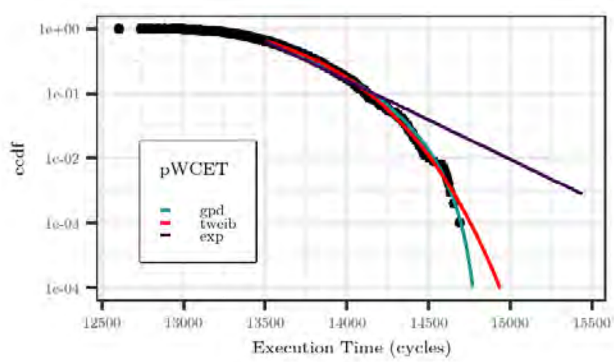

Fig. 1: Sample of $n = 1000$ of railway case study data with different pWCET fittings

into probability language means that as the time passes when a program is executing, the probability for it to finish gets smaller. Now naturally, these kinds of probability distributions are under the label of light tails, but as we mentioned they deliver optimistic tail distributions. In risk analysis, there is also a property that describes the CRTES program behaviour, and that is Increasing Hazard Rate (IHR). We worked searching by an alternative solution to light tails within the risk analysis domain, and drew en equivalence between IHR and non-heavy tails. From there we derived the next theorem. In order to use IHR distributions for pWCET estimation, we build upon the following theorem proven in [12] and [13]: *Theorem.* Given a non-negative random variable X, with f and F the pdf and cdf, respectively (where $H(x) = -\log(1 - F(x)), \quad x \in support(X)$),

$$\log(f) \text{ concave} \Rightarrow X \text{ IHR} \Leftrightarrow H \text{ convex} \tag{1}$$

From here, we found that a function that satisfies all these properties is the *tailW*. The *tailW* law is constructed using the excess probability function. Thus, the cdf is $F(x, \alpha, \beta, \nu) = 1 - \exp\big(-\alpha(x + \nu)^{\beta} + \alpha\nu^{\beta}\big)$ for $x \geq 0$, $\alpha > 0$, $\beta \geq 0$ and $\nu > 0$. Now we can see in Figure 1 we see the complementary cumulative distribution function of the execution times of a program, which represents the probability of having another value bigger than a given execution time. We see how the *tailW* performs better than the exponential function for a small sample of $n = 1000$, and slightly worse than the gpd with light tails. This sample was drawn from a bigger railway case study data of $n = 10^7$. Now, if we take the models with the parameters resulting from fitting the sample of $n = 1000$, and use them to predict the whole data, we will see how *tailW* performs against the others. In Figure 2 we see how *tailW* is close to the real values of the distribution, while not being overly pessimistic as the exponential function, but also not optimistic and falling behind the data as the gpd with light tails.

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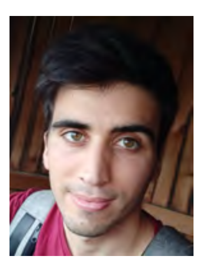

Sergi Vilardell received his BSc degree in Physics from Universitat Autonoma de Barcelona (UAB), in 2016. The following year, he completed his MSc degree in Modelling for Science and Engineering from UAB in 2017. Since then, he has been working with Isabel Serra from Centre de Recerca Matemàtica (CRM) in Extreme Value Theory problems. Since 2018 he works with the Computer Architecture and Operating Systems (CAOS) group at Barcelona Supercomputer Center (BSC), as well as a PhD student at the department of computer architecture of Universitat Politècnica de Catalunya (UPC), Spain.